

# Stability of 1+1 dimensional causal relativistic viscous hydrodynamics

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## Abstract

The stability of the 1+1 dimensional solution of Israel-Stewart theory is investigated. Firstly, the evolution of the temperature and the ratio of the bulk pressure over the equilibrium pressure of the background is explored. Then the stability with linear perturbations is studied by using the Lyapunov direct method. It shows that the shear viscosity may weaken the instability induced by the large peak of bulk viscosity around the phase transition temperature  $T_c$ .

*Keywords:* Israel-Stewart theory, stability, viscosity

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## 1. Introduction

A hot and dense partonic matter has been created at the Relativistic Heavy Ion Collider (RHIC) of Brookhaven National Laboratory [1], which is thought to be a kind of perfect liquid of quark and gluon [2]. The study on the properties of this matter is a hot issue. Presently one property which is of very interest is the exact value of the ratio of viscosity over entropy density for the matter. To extract the value, one needs to compare experimental data with relativistic viscous hydrodynamics simulation [3, 4, 5, 6, 7].

Two different kinds of relativistic viscous hydrodynamics have been developed so far. One is the so-called first order relativistic viscous hydrodynamics which was first developed by Eckart and varied by Landau and Lifshitz[8, 9], and many calculations have been done since then[10]. But there are two problems in the approaches: one is that dissipative fluctuation may propagate at a speed larger than the speed of light and thus leads to a causality problem; the other is that the solution which may develop instabilities [11]. These problems have been studied extensively in Refs. [12, 13]. The other is the second order theories among which the Israel-Stewart theory is popularly used so far [14]. However, before Israel-Stewart theory is applied to describe heavy ion collisions, one should know whether it is stable or not. Till now, there have been some stability analysis on the issue [15, 16, 17, 18]. In [15, 16, 17, 18], the authors used the plane wave perturbation method to study the stability of the theory around a hydrostatic state and discussed the regime of validity of hydrodynamics. Also in [17], using the Lyapunov direct method as in [13], the authors analyzed the stability of the scaling solution with only bulk viscosity and presented the stable regions of the scaling solu-

tion for both homogeneous and inhomogeneous perturbations. And several groups have compared this theory with the experimental data and show that this theory can be used without a problem when the shear viscosity and bulk viscosity are small [3, 4, 5, 6, 7, 19].

It is well known that hydrodynamics fails to describe the HBT results, i.e, the calculated ratio of HBT radii in outward direction over that in side-ward direction is higher than the experimental data[20]. Recent attempt to reconcile them is to take bulk viscosity into account. But in [20, 21, 22], the authors found that the HBT radii hardly change if the peak of bulk viscosity is small and the other parameters such as the equation of state, initial condition and so on keep unchanged. However, in [23] the authors analyzed the stability of Navier-Stokes theory with the bulk viscosity which has a large peak around  $T_c$  and found that there exists some inhomogeneous modes which will tear the system into droplets. If this is the case, it will help us resolving the interferometric data[24]. But there are some basic problems with Navier-Stokes theory as mentioned above and the authors did not consider the effects of shear viscosity additionally. In this paper we use the same method to study if this could happen in the Israel-Stewart theory and what is the role of the shear viscosity.

## 2. Israel-Stewart theory and linear perturbations

The general hydrodynamic equations arise from the local conservation of energy and momentum [25]

$$\partial_\mu T^{\mu\nu}(x) = 0, \tag{1}$$

where the energy-momentum tensor without heat conduction is decomposed into the following form [7]

$$T^{\mu\nu} = eu^\mu u^\nu - (p+\Pi)\Delta^{\mu\nu} + \pi^{\mu\nu}. \quad (2)$$

where  $e$  and  $p$  are the local energy density and thermal equilibrium pressure, and  $u^\mu$  is the 4-velocity of the energy flow which obeys  $u^\mu u_\mu = 1$ .  $\Pi$  is the bulk viscous pressure, and  $\Delta^{\mu\nu} = g^{\mu\nu} - u^\mu u^\nu$  is transverse to the flow velocity, that is  $\Delta^{\mu\nu} u_\nu = 0$ .  $\pi^{\mu\nu}$  is the traceless shear viscous pressure tensor. With Eq. (1) and Eq. (2), we can get the evolution equations of the energy density and the 4-velocity of energy flow.

In Israel-Stewart approach, the kinetic evolution equations of the bulk pressure  $\Pi$  and the traceless shear viscous tensor  $\pi^{\mu\nu}$  are [7]

$$D\Pi = -\frac{1}{\tau_\Pi}(\Pi + \zeta\nabla\cdot u), \quad (3)$$

$$D\pi^{\mu\nu} = -\frac{1}{\tau_\pi}(\pi^{\mu\nu} - 2\eta\langle\nabla^\mu u^\nu\rangle) - (u^\mu\pi^{\nu\alpha} + u^\nu\pi^{\mu\alpha})Du_\alpha, \quad (4)$$

where  $D$  is the time derivative in the local fluid rest frame and fulfills  $D = u^\mu\partial_\mu$ . The angular bracket notation is defined by  $\langle\nabla^\mu u^\nu\rangle = \frac{1}{2}(\nabla^\mu u^\nu + \nabla^\nu u^\mu) - \frac{1}{3}(\nabla\cdot u)\Delta^{\mu\nu}$ .  $\eta$  and  $\zeta$  denote the bulk and shear viscous coefficient, respectively.  $\tau_\pi$  and  $\tau_\Pi$  are the relaxation time for the bulk pressure and the shear tensor, respectively. They can be related to  $\eta$  and  $\zeta$  as follows [11]

$$\tau_\pi = 2\eta\beta_2, \quad \tau_\Pi = \zeta\beta_0. \quad (5)$$

where  $\beta_2$  and  $\beta_0$  are the relaxation coefficients that need to be calculated from other theories.

When hydrodynamics is applied to describe heavy ion collisions, people always use the symmetry of the collision system to simplify the evolution equations. Thus we have the scaling solution, the 1+1 solution and the 2+1 solution [26, 3, 4, 5]. Here we focus on the 1+1 solution of the Israel-Stewart theory. For 1+1 solution, the system has boost-invariant longitudinal expansion and transverse expansion in one dimension. For these geometries, it is convenient to work in the co-moving and radial coordinates  $\tau, r, \phi, \eta$ . We consider a small perturbation, and the backgrounds  $\epsilon_0(\tau), \pi_{\nu 0}^\mu(\tau), \Pi_0(\tau)$  evolve with time

$$\epsilon(\tau, r) = \epsilon_0(\tau) + \delta\epsilon(\tau, r), \quad (6)$$

$$v(\tau, r) = \delta v(\tau, r), \quad (7)$$

$$\pi_\nu^\mu(\tau, r) = \pi_{\nu 0}^\mu(\tau) + \delta\pi_\nu^\mu(\tau, r), \quad (8)$$

$$\Pi(\tau, r) = \Pi_0(\tau) + \delta\Pi(\tau, r). \quad (9)$$

With Eqs. (1~4) and Eqs. (6~9) the evolution equations of backgrounds can be given as follows

$$\partial_\tau \epsilon_0 = -(\epsilon_0 + p_0 + \Pi_0) \frac{1}{\tau} + \pi_{\eta 0}^\eta \frac{1}{\tau}, \quad (10)$$

$$\tau_{\pi 0} \partial_\tau \pi_{\eta 0}^\eta + \pi_{\eta 0}^\eta = \frac{4}{3} \frac{\eta_0}{\tau}, \quad (11)$$

$$\tau_{\pi 0} \partial_r \pi_{r 0}^r + \pi_{r 0}^r = -\frac{2}{3} \frac{\eta_0}{\tau}, \quad (12)$$

$$\tau_{\Pi 0} \partial_\tau \Pi_0 + \Pi_0 = -\frac{\zeta_0}{\tau}. \quad (13)$$

We can also get the evolution equations of the perturbations

$$\begin{aligned} & [(\epsilon_0 + p_0 + \Pi_0 - \pi_{r0}^r) \partial_\tau + \partial_\tau (p_0 + \Pi_0) - (\partial_\tau + \frac{1}{\tau}) \pi_{r0}^r + \frac{1}{\tau} \pi_{\eta 0}^\eta] \delta v \\ & + \partial_r (\delta p + \delta \Pi) - (\partial_r + \frac{2}{r}) \delta \pi_r^r - \frac{1}{r} \delta \pi_\eta^\eta = 0, \end{aligned} \quad (14)$$

$$\begin{aligned} & [(\epsilon_0 + p_0 + \Pi_0) (\partial_r + \frac{1}{r}) - \pi_{r0}^r (\partial_r - \frac{1}{r}) + \frac{1}{r} \pi_{\eta 0}^\eta] \delta v + \partial_\tau \delta \epsilon \\ & + \frac{1}{\tau} (\delta \epsilon + \delta p + \delta \Pi) - \frac{1}{\tau} \delta \pi_\eta^\eta = 0, \end{aligned} \quad (15)$$

$$\tau_{\pi 0} \partial_\tau \delta \pi_\eta^\eta + \delta \pi_\eta^\eta = -\frac{\delta \tau_\pi}{\tau_{\pi 0}} \left( \frac{4}{3} \frac{\eta_0}{\tau} - \pi_{\eta 0}^\eta \right) - \frac{2}{3} \eta_0 \left( \partial_r \delta v + \frac{\delta v}{r} \right) + \frac{4}{3} \frac{\delta \eta}{\tau}, \quad (16)$$

$$\tau_\pi \partial_\tau \delta \pi_r^r + \delta \pi_r^r = \frac{\delta \tau_\pi}{\tau_{\pi 0}} \left( \frac{2}{3} \frac{\eta_0}{\tau} + \pi_{r0}^r \right) - \frac{2}{3} \eta_0 \left( -2 \partial_r \delta v + \frac{\delta v}{r} \right) - \frac{2}{3} \frac{\delta \eta}{\tau}, \quad (17)$$

$$\tau_{\Pi 0} \partial_\tau \delta \Pi + \delta \Pi = \frac{\delta \tau_\Pi}{\tau_{\Pi 0}} \left( \Pi_0 + \frac{\zeta_0}{\tau} \right) - \zeta_0 \left( \partial_r \delta v + \frac{\delta v}{r} \right) - \frac{\delta \zeta}{\tau}. \quad (18)$$

In order to get rid of the space-like derivatives, we can do the Hankel transform (Fourier-Bessel transform) due to the geometry system we use here[3].

After introducing

$$\delta \tilde{\pi} = \left( \partial_r + \frac{2}{r} \right) \delta \pi_r^r + \frac{1}{r} \delta \pi_\eta^\eta \quad (19)$$

we do the following Hankel transforms,

$$\delta v(\tau, r) = \int_0^\infty dk J_1(kr) k \delta \tilde{v}(\tau, k), \quad (20)$$

$$\delta \epsilon(\tau, r) = \int_0^\infty dk J_0(kr) k \delta \tilde{\epsilon}(\tau, k), \quad (21)$$

$$\delta\pi(\tau, r) = \int_0^\infty dk J_1(kr) k \delta\tilde{\pi}(\tau, k), \quad (22)$$

$$\delta\pi_\eta^\eta(\tau, r) = \int_0^\infty dk J_0(kr) k \delta\tilde{\pi}_\eta^\eta(\tau, k), \quad (23)$$

$$\delta\Pi(\tau, r) = \int_0^\infty dk J_0(kr) k \delta\tilde{\Pi}(\tau, k), \quad (24)$$

where  $k$  is the wave number. It stands for a homogeneous perturbation when  $k$  equals zero or an inhomogeneous perturbation when  $k$  is not zero. Its range for a realistic QGP fluid has been roughly estimated in [13]. With Eqs. (14~24), we can get the evolution equations for  $\delta\tilde{v}, \delta\tilde{\epsilon}, \delta\tilde{\pi}_\eta^\eta, \delta\tilde{\pi}, \delta\tilde{\Pi}$

$$\begin{aligned} [(\epsilon_0 + p_0 + \Pi_0 - \pi_{r0}^r)\partial_\tau + \partial_\tau(p_0 + \Pi_0) - (\partial_\tau + \frac{1}{\tau})\pi_{r0}^r + \frac{1}{\tau}\pi_{\eta 0}^\eta]\delta\tilde{v} \\ - kc_s^2\delta\tilde{\epsilon} - k\delta\tilde{\Pi} - \delta\tilde{\pi} = 0, \end{aligned} \quad (25)$$

$$(\epsilon_0 + p_0 + \Pi_0 + \frac{1}{2}\pi_{\eta 0}^\eta)\delta\tilde{v} + (\partial_\tau + \frac{1+c_s^2}{\tau})\delta\tilde{\epsilon} + \frac{1}{\tau}\tilde{\Pi} - \frac{1}{\tau}\delta\tilde{\pi}_\eta^\eta = 0, \quad (26)$$

$$\begin{aligned} \frac{2}{3}\frac{\eta_0}{\tau_{\pi 0}}k\delta\tilde{v} + \frac{1}{\tau_{\pi 0}^2}(\frac{4}{3}\frac{\eta_0}{\tau} - \pi_{\eta 0}^\eta)(\frac{\partial\tau_\pi}{\partial\epsilon})_0\delta\tilde{\epsilon} - \frac{4}{3}\frac{1}{\tau\tau_{\pi 0}}(\frac{\partial\eta}{\partial\epsilon})_0\delta\tilde{\epsilon} \\ + (\partial_\tau + \frac{1}{\tau_{\pi 0}})\delta\tilde{\pi}_\eta^\eta = 0, \end{aligned} \quad (27)$$

$$\begin{aligned} \frac{4}{3}\frac{\eta_0}{\tau_{\pi 0}}k^2\delta\tilde{v} - \frac{1}{2}\frac{1}{\tau_{\pi 0}^2}(\frac{4}{3}\frac{\eta_0}{\tau} - \pi_{\eta 0}^\eta)(\frac{\partial\tau_\pi}{\partial\epsilon})_0k\delta\tilde{\epsilon} + \frac{2}{3}\frac{1}{\tau\tau_{\pi 0}}(\frac{\partial\eta}{\partial\epsilon})_0k\delta\tilde{\epsilon} \\ + (\partial_\tau + \frac{1}{\tau_{\pi 0}})\delta\tilde{\pi} = 0, \end{aligned} \quad (28)$$

$$\frac{\zeta_0}{\tau_{\Pi 0}}k\delta\tilde{v} - \frac{1}{\tau_{\Pi 0}^2}(\frac{\zeta_0}{\tau} + \Pi_0)(\frac{\partial\tau_\Pi}{\partial\epsilon})_0\delta\tilde{\epsilon} + \frac{1}{\tau\tau_{\Pi 0}}(\frac{\partial\zeta}{\partial\epsilon})_0\delta\tilde{\epsilon} + (\partial_\tau + \frac{1}{\tau_{\Pi 0}})\delta\tilde{\Pi} = 0. \quad (29)$$

### 3. Results

In order to get numerical results, the values of viscosities and the relaxation time are set up as described below. As to shear viscosity, the strong coupling theory and the hydrodynamic and transport model show that  $\frac{\eta}{s}$  can not be too large [27, 4, 28]. Here we use two different values  $\frac{\eta}{s} = 0.02$  and 0.2 to see its effects. The relaxation time of the shear viscosity is set to the Boltzmann gas result  $\tau_\pi = \frac{\eta}{s} \frac{6}{T}$  in our work. Although recent results of SU(3) Yang-Mills theory show that the effects of a potentially large bulk viscosity near  $T_c$  are more subtle to detect in spectral integrals[29] than previous work suggested [30], but there are still some possibilities that  $\frac{\zeta}{s}$  becomes large around  $T_c$ [31]. So the following parametrization as in [23] is adopted to see the effects of bulk viscosity on the stability

$$\zeta = s(z_{pQCD} + \frac{z_0}{\sqrt{2\pi}\sigma}\exp[-\frac{t^2}{2\sigma^2}]), \quad (30)$$

where  $t = T - T_c$ ,  $\sigma = 0.01T_c$ ,  $z_{pQCD} \sim 10^{-3}$  and different  $z_0$  denotes different magnitudes of  $\zeta/s$ . For the relaxation time of bulk viscosity, the parametrization similar to the relaxation time of shear viscosity is employed

$$\tau_\Pi = b \frac{\zeta}{\epsilon + p}, \quad (31)$$

where different  $b$  is used to see the effect of relaxation time of bulk viscosity.

To connect to relativistic heavy-ion collisions, the equation of state in [32] is adopted and the initial temperature  $T_0$  is set to be 0.34 GeV. Firstly, let us see the effect of viscosity on the evolution of background. Fig 1. shows the profiles of background temperature for different  $\eta/s$  and for different  $z_0$  and  $b$ , corresponding to different magnitudes and relaxation times of bulk



viscosity. Each plot consists of two different  $\eta/s$ . From the left to the right, the plots represent  $z_0 = 0.0$  and  $0.1 T_c$ , respectively. These plots show that the shear viscosity can slow down the evolution of the source. We can also see that when  $b$  is large which means that the relaxation time of bulk viscosity is large, bulk viscosity hardly have effects on the evolution of temperature even when  $\frac{\zeta}{s}$  has a large peak. But when the relaxation time is small, for a larger magnitude of bulk viscosity, we can see that the system stays at a nearly constant temperature which is the same as the case found in Navier-Stokes theory in [23]. This effect is not hard to be understood. Because the role of the relaxation time is to delay the appearance of viscous forces [24], bulk viscosity has effects on the evolution of the source only after a time-scale  $\tau_{\Pi}$ . The larger the relaxation time is, the longer time the bulk viscosity needs to affect on the evolution of the system. Meanwhile, the system is still evolving and it may be cooled down to be below  $T_c$  where bulk viscosity can be negligible. So if the relaxation time is large enough and the bulk viscosity has a small width, bulk viscosity may have negligible effects on the evolution of background. It is consistent with the results in [22]. In [22] we found that the width of bulk viscosity has larger effects on the evolution of the source than the magnitude of bulk viscosity.

Fig 2. shows the profiles of the ratio of bulk pressure over the equilibrium pressure of the background for different cases as in Fig 1. We can see that  $\Pi/p$  has a peak when  $\zeta/s$  has a peak around  $T_c$ . And the fact that  $\Pi/p$  exceeds one means that the state is far away from equilibrium. It indicates that the matter is not only hydrodynamically unstable, but also thermodynamically unstable as stated in [26] when the peak of bulk viscosity is large

and the relaxation time of bulk viscosity is small. We can also see that shear viscosity will decrease  $\Pi/p$  which indicates that shear viscosity will weaken the instability that induced by bulk viscosity. This behavior is also found in the next analysis about linear perturbations.

Now we use Lyapunov direct method to study the stability of 1+1 solution of Israel-Stewart theory. The evolution equations of perturbations Eqs. (25~29) can be rewritten in the following matrix formula

$$\partial_\tau \delta Y = A \delta Y, \quad (32)$$

where

$$\delta Y = \begin{pmatrix} \delta \tilde{v} \\ \delta \tilde{\epsilon} \\ \delta \tilde{\pi}_\eta^\eta \\ \delta \tilde{\pi} \\ \delta \tilde{\Pi} \end{pmatrix}. \quad (33)$$

When one uses the Lyapunov direct method, the Lyapunov function should be given first. The Lyapunov function must be positive definite. If it is a monotonically decreasing function, then the solution is stable; if it is a monotonically increasing function, then the solution is unstable. The more detailed description about this method can be found in [13, 17]. Here we assume the Lyapunov function is  $V = \delta Y^T \delta Y$ . Then the evolution equation of the Lyapunov function  $V$  is

$$\partial_\tau V = \delta Y^T (A^T + A) \delta Y, \quad (34)$$

after a short derivation, we can get

$$\lambda_{min} V \leq \partial_\tau V \leq \lambda_{max} V \quad (35)$$

where  $\lambda_{\max}$  and  $\lambda_{\min}$  are the largest and smallest eigenvalues of  $A + A^T$ , respectively. It can be shown that the solution is stable when  $\lambda_{\max} \leq 0$  and is unstable when  $\lambda_{\min} \geq 0$ . Fig. 3 and Fig. 4 show the values of  $\lambda_{\max}$  and  $\lambda_{\min}$  for  $k = 0$  and  $k = 3$ , respectively. Each figure has different cases as stated for Fig. 1. We can see that neither the stable region nor the unstable region can be determined because  $\lambda_{\max}$  is always larger than zero and  $\lambda_{\min}$  is always smaller than zero. As for the scaling solution, the unstable regions also can not be found but the stable regions can be determined [17]. These plots show that  $\lambda_{\max}$  and  $\lambda_{\min}$  have rapid change when the bulk viscosity is large and the relaxation time is small. We can also see that shear viscosity will delay this kind of rapid change and reduce the magnitude of  $\lambda_{\max}$  and  $\lambda_{\min}$ . This phenomenon is the same for different  $k$ , but the magnitude of  $\lambda_{\max}$  and  $\lambda_{\min}$  is larger with larger  $k$ .

We can see from Fig. 3 and Fig. 4 that the absolute value of  $\lambda_{\max}$  and  $\lambda_{\min}$  increase rapidly which means the growing and damping rates of perturbations increase rapidly due to the large peak and small relaxation time of bulk viscosity. Therefore the perturbations may rapidly grow to a value comparable with the background. So they will break local homogeneity and play an important role in the subsequent evolution of the system. The created inhomogeneities have no reinteractions. It is possible that isolated fragments will be created and move away from each other. In [23], the authors argued that this may be a reason that the source will be clusterized and then decoupled as the fireballs. By adding a further free parameter which is the cluster size to the system, this mechanism may solve the HBT problem[24]. In the Israel-Stewart theory this phenomenon may also happen when the

peak of  $\zeta/s$  is large,  $\tau_\Pi$  is small and  $\zeta/s$  is not too large.

#### 4. Conclusions

In summary, the stability problem of the 1+1 solution of Israel-Stewart theory is studied. Firstly, the evolution of the temperature and the ratio of the bulk pressure over the equilibrium pressure are studied. We find that both shear and bulk viscosity slow down the evolution of temperature. And the ratio of bulk pressure over the equilibrium pressure ( $\Pi/p$ ) will exceed one with a large peak of bulk viscosity, which indicates the state is unstable. The shear viscosity reduces the magnitude of  $\Pi/p$  to weaken the effects of bulk viscosity. Then using Lyapunov direct method, we can not determine the stable or unstable regions. We also find the phenomenon which may drive the source to clusterize which is similar to that in Navier-Stokes theory. However, this phenomenon will happen only when the peak of bulk viscosity is large enough, the relaxation time of the bulk viscosity is small and the shear viscosity is not too large.

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Figure 1: Evolutions of background temperature for different shear and bulk viscosity sets. See texts for details.

Figure 2:  $\Pi/p$  as a function of  $T/T_c$  for different shear and bulk viscosity sets. See texts for details.

Figure 3:  $\lambda_{max}$  and  $\lambda_{min}$  for  $k = 0$  with different shear and bulk viscosity sets. See texts for details.

Figure 4:  $\lambda_{max}$  and  $\lambda_{min}$  for  $k = 3$  with different shear and bulk viscosity sets. See texts for details.



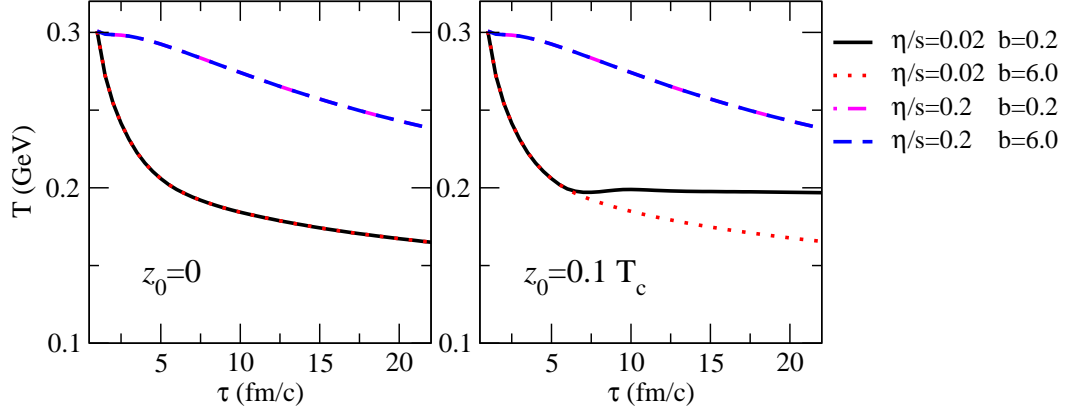


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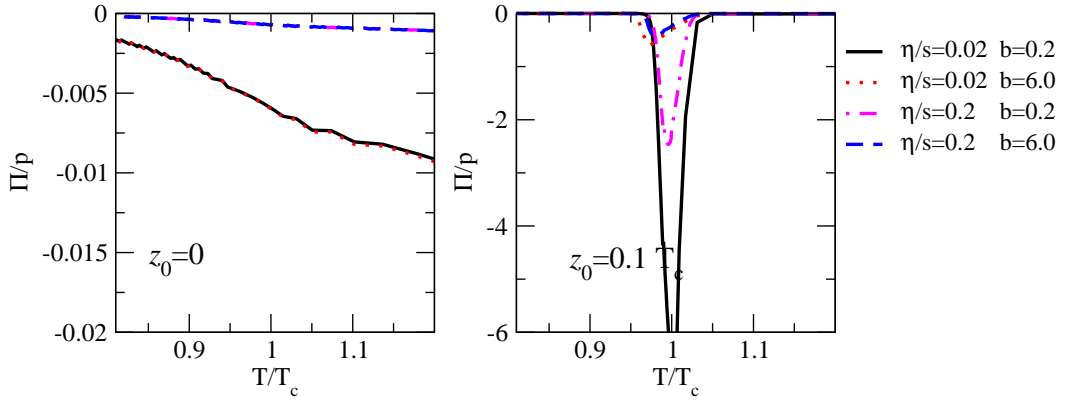


Figure 2:  $\Pi/p$  as a function of  $T/T_c$  for different shear and bulk viscosity sets. See texts for details.

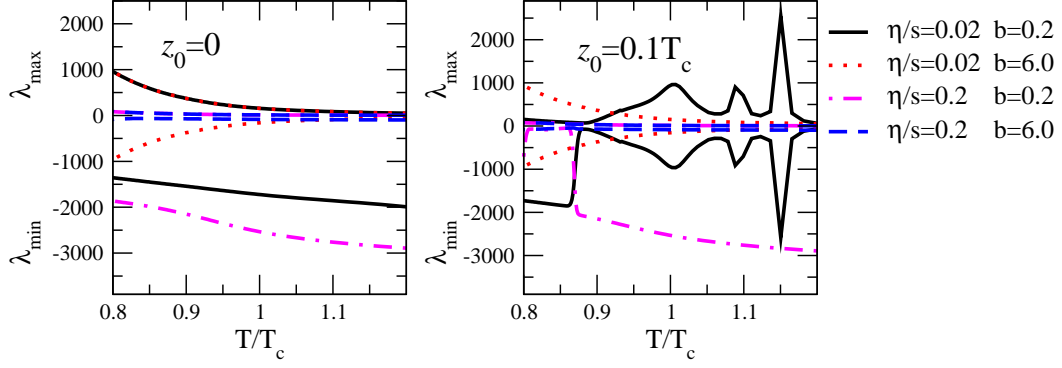


Figure 3:  $\lambda_{\max}$  and  $\lambda_{\min}$  for  $k = 0$  with different shear and bulk viscosity sets. See texts for details.

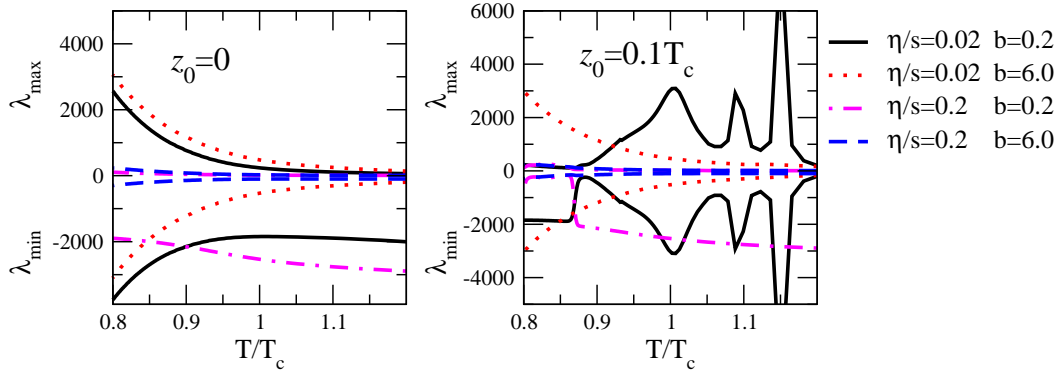


Figure 4:  $\lambda_{\max}$  and  $\lambda_{\min}$  for  $k = 3$  with different shear and bulk viscosity sets. See texts for details.